On the Velocity of Rotation of the Electric Discharge in Gases at Low Pressures in a Radial Magnetic Field.

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The following paper contains an account of a series of experiments on the motion of an electric discharge in a magnetic field perpendicular to the direction of the discharge current. The fact that the discharge moves in a magnetic field like a flexible conductor carrying a current was discovered long ago, and De La Rive showed that it could be made to rotate continuously round one pole of a magnet placed inside the vacuum tube.

The apparatus used in the present experiments was similar in principle to De La Rive's, but was arranged so that fairly exact measurements of the various quantities concerned could be obtained. Fig. 1 shows a vertical section of the vacuum tube and magnet used. The tube consisted of two concentric glass tubes cemented with sealing wax into aluminium discs. The discs had grooves turned in them to fit the glass tubes, and the part of the discs between the tubes projected a few millimetres, so that there was no danger of the discharge passing through the sealing wax. Polished platinum rings were fixed on to the aluminium discs between the glass tubes, and these formed the electrodes between which the discharge was passed. The ends of the tubes were carefully ground truly perpendicular to their axes, and the two platinum rings were accurately parallel. To keep the electrodes cool, a ring of narrow brass tubing was soldered on to the back of each disc and a stream of water was kept flowing through these whenever a discharge was passed. This arrangement enabled comparatively large currents to be used without softening the sealing wax. A narrow copper tube was soldered into one of the discs and communicated with the interior of the vacuum tube through a fine hole. The tube was connected by a mercury sealed joint with a glass tube leading to a bulb containing pure phosphorus pentoxide, a Töpler pump, and a McLeod gauge.

The aluminium discs could be connected through an ammeter and resistance to a battery of 800 secondary cells. The magnet used to produce the magnetic field consisted of a soft iron bar which was magnetised by means of two solenoids, one at each end, so as to have one pole in the middle and opposite poles at each end. The bar was fixed along the axis of the vacuum tube as shown, so that the middle of the bar was at the centre of the tube. The bar was

surrounded by a closely fitting thin-walled glass tube, which prevented the aluminium discs coming in contact with it.

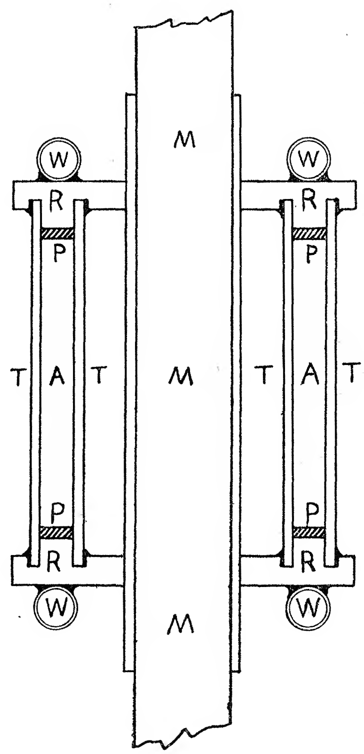

Fig. 1.—Vertical section of discharge tube through its axis.

M M M—Iron bar with glass tube round it.
R R R R—Aluminium rings.
P P P P—Platinum rings.
T T T T—Glass tubes.
A A—Annular space through which discharge passed.
W W W W—Water cooling tubes.

The distribution of the magnetic force around the bar in the region occupied by the vacuum tube was investigated by means of a small compass needle and was found to be sensibly radial.

The current through the magnetising solenoids was measured with an accurate ammeter. Before beginning an experiment, the bar was demagnetised by the method of reversals and then the current was slowly increased, with frequent reversals, up to the required value. In this way the effects of permanent magnetism in the bar were eliminated and the magnetic field could be regarded as a definite function of the current.

The strength of the magnetic field corresponding to different values of the current was found by means of a coil of 50 turns of wire, which was

connected to a ballistic galvanometer and placed round the bar magnet in the position usually occupied by the vacuum tube. The deflections of the galvanometer, produced by reversing a series of currents in the solenoids, were found, and then the coil was placed in a solenoid giving a field of known strength, which was then reversed and the galvanometer deflection measured. The coil of 50 turns was wound in two sections in grooves of square cross-section turned on a boxwood circular cylinder, each groove containing 25 turns. The outside diameter of each section was equal to the outside diameter of the annular space between the two glass tubes forming the vacuum tube, and the inside diameters were equal to the inside diameter of the annular space. The distance between the centres of the two sections was equal to the distance between the platinum electrodes. The two sections were first connected together in series, so that a current would pass round them in opposite directions, and the boxwood cylinder was then placed in the position of the vacuum tube on the iron bar and the galvanometer deflections obtained. The two sections were then connected in series, so that a current would pass round them in the same direction, and were placed inside the solenoid so that the axis of the cylinder was parallel to its magnetic field, and the galvanometer deflection due to reversing this field was obtained.

Let S = strength of field due to the solenoid when a current of 1 ampère is passed round it.
C = current reversed in the solenoid.
r = radius of the mean area of each section of the coil of 50 turns.
l = distance between the centres of the two sections.
d = galvanometer swing due to reversing C.
d' = galvanometer swing due to reversing the magnetisation of the iron bar.
F = field due to iron bar at radius r.

Then $$\mathrm{A}d = 100\,\pi r^2\mathrm{SC} \quad \text{and} \quad \mathrm{A}d' = 100\,\pi r l\mathrm{F},$$

where A is a constant. Hence

$$\mathrm{F} = \mathrm{SC}rd'/ld.$$

The values of F corresponding to a series of currents in the solenoids for magnetising the bar were found in this way.

The following are the dimensions of the vacuum tube:—

Inside diameter of outside tube	3·01 cm.
Outside diameter of inside tube	2·32 „
Distance between electrodes	3·17 „

The radius of the mean area of each section of the coil used to find the strength of the magnetic field was 1·34 cm., which is nearly equal to the mean radius, 1·33 cm. The magnetic field acting on the discharge is inversely proportional to the radius, so that the parts of the discharge furthest from the axis of the tube are acted on by the weakest field. The fields given in the tables of results are in every case the field at the radius $r = 1{\cdot}34$ cm.

When a discharge was passed through a gas at a few millimetres pressure it usually formed a narrow positive column perpendicular to the surfaces of the electrodes. The negative glow covered the part of the negative electrode opposite to the end of the positive column and extended over a greater or less area besides, according to the strength of the current. With large currents, the whole of the negative electrode was covered by the negative glow, but even when this was the case the positive column remained quite narrow, usually not more than 1 cm. wide. At low pressures with large currents, the positive column sometimes spread out and extended all round the tube, so that it became impossible to observe the rotation of the discharge when a magnetic field was produced.

When the iron bar was magnetised so as to produce a radial magnetic field through the vacuum tube, the positive column usually began to move rapidly round the annular space between the electrodes. The negative glow went round with the positive column unless it covered the whole surface of the electrode, in which case it was impossible to tell whether it went round or not.

The number of revolutions per second was found by observing the discharge through a stroboscopic disc driven by an electric motor. The speed of rotation of the disc was measured with a revolution counter, which was usually kept on for 30 seconds at a time. The disc had several concentric rings of holes bored in it and its speed was adjusted so that the discharge as seen through it appeared to remain at rest in one of the circles of holes. In this way it was usually easy to get the number of revolutions per second made by the discharge in terms of the number made by the disc. With a weak magnetic field the rotation was sometimes slow enough for the revolutions to be directly counted.

At low pressures with strong fields the speed sometimes was several hundred revolutions per second, and it was rather difficult to be quite sure of the relation between the speed of the disc and the speed of the discharge. In such cases several independent determinations were made, using different speeds for the disc. When the way in which the speed depends on the magnetic field and gas pressure had been found, it was possible to calculate roughly what the speed might be expected to be in a particular case, so that

it became easy to find the correct factor required to convert the speed of the disc into the speed of the discharge.

When a weak magnetic field was turned on, the discharge sometimes did not start revolving, but appeared to stick at one of the electrodes (usually the cathode) and was bent into the shape of a screw round the discharge tube. The screw usually did not make more than a fraction of a turn round the tube. On increasing the field slowly, the discharge suddenly began to rotate, and then, on diminishing the field, would continue rotating with a much smaller field than was necessary to start it. When rotating rapidly, the discharge as seen in the stroboscope appeared exactly as it appeared when at rest without any field, and was always perpendicular to the electrodes. An exception to this rule was observed in the case of hydrogen at low pressures, when the positive column sometimes became broader when set rotating and sometimes split up into two separate columns diametrically opposite each other, which always coalesced on turning off the field.

The rotation of the discharge in air, nitrogen, and hydrogen was examined. The air was passed over solid caustic potash and dried with phosphorus pentoxide. The nitrogen was prepared by the action of pure urea on potassium hypobromite and was passed over caustic potash, calcium chloride, and phosphorus pentoxide; it appeared to be pure and did not act on the mercury in the pump. The hydrogen was prepared by the action of pure hydrochloric acid on pure zinc, and was passed over caustic potash, calcium chloride, and phosphorus pentoxide. The spectrum of the discharge was observed in each case through a large direct-vision spectroscope and appeared to be that of the gas supposed to be present. The most complete set of results was obtained with nitrogen; hydrogen was found difficult to work with, except at high pressures.

The Rotation of the Discharge in Nitrogen.

The following table (page 422) gives some of the results obtained, showing the variation of the velocity of rotation with the magnetic field at several pressures.

Magnetic field. H.	Revolutions per sec. n.	n/H.	Magnetic field. H.	Revolutions per sec. n.	n/H.
Pressure 10·3 mm.			Pressure 4·35 mm.		
41	9·5	0·23	27	11·5	0·43
59	13·9	0·24	46	20·0	0·44
74	19·0	0·26	51	27·0	0·53
96	23·7	0·25	59	32·0	0·54
121	31·3	0·26	73	43·0	0·59
134	35·9	0·27	88	59·0	0·67
			109	88·0	0·81
		Mean 0·25			Mean 0·57
Pressure 7·0 mm.			Pressure 11·4 mm.		
41	14·9	0·36	31	6·4	0·21
59	20·5	0·35	40	9·0	0·22
74	26·9	0·36	78	16·1	0·21
98	37·8	0·38	107	25·0	0·23
			133	32·0	0·24
		Mean 0·36			Mean 0·22

It will be seen that, except at the lowest pressure, the velocity of rotation at each pressure is nearly proportional to the magnetic field.

In the following table, the mean values of n/H are given, and the product of n/H and the pressure:—

Pressure p.	n/H.	np/H.
11·4	0·22	2·50
10·3	0·25	2·57
7·0	0·36	2·52
4·35	0·57	2·48

The product np/H is nearly constant, so that the number of revolutions per second (n) is given approximately by the equation $n = \mathrm{BH}/p$, where $\mathrm{B} = 2·5$.

In all the above experiments the currents carried by the discharge were between 10^{-2} and 3×10^{-2} ampère. A number of measurements were made to find how the velocity varied with the current. The results obtained can be best exhibited by means of curves. Fig. 2 shows a typical curve representing a series of observations at constant pressure and with a constant magnetic field.

It will be seen that the velocity of rotation passes through a very flat minimum value as the current is varied. The numbers given above on the variation of the velocity with the pressure and the field are for currents lying in the range for which the velocity is nearly independent of the

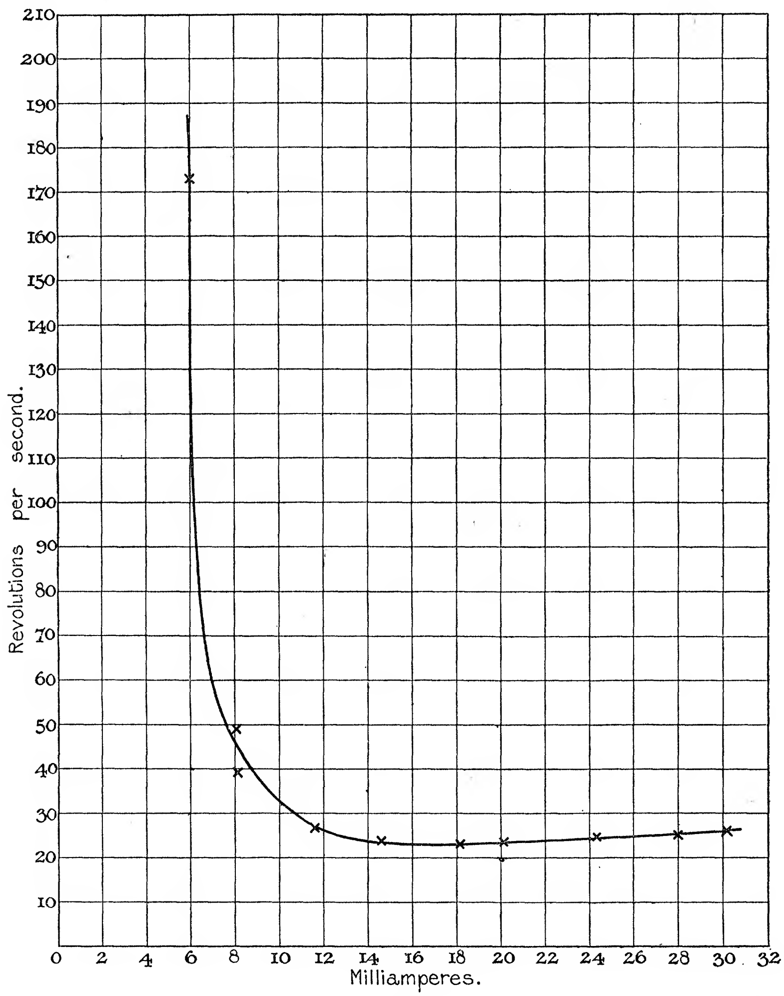

Fig. 2.—Magnetic field 41. Pressure 2·8 mm.

current. The velocity appeared to be nearly proportional to the magnetic field under any conditions, but when the velocity varied rapidly with the current it was difficult to obtain satisfactory measurements, because of the difficulty of keeping the current exactly constant.

The Velocity of Rotation in Air and in Hydrogen.

Similar results were obtained with air to those obtained with nitrogen. The following table contains some of them :—

p.	n.	H.	np/H.
4·7	19·5	34	2·7
4·7	42·2	78	2·5
8·0	14·7	40	2·9
8·0	29·1	85	2·7
11·8	22·1	88	3·0
			Mean 2·8

It will be seen that np/H is nearly constant, so that, for air, $n = 2{\cdot}8\mathrm{H}/p$ and so does not differ much from n in nitrogen. The rapid variation of n with the current observed with nitrogen when the current was small was not observed with air.

In the case of hydrogen, the ranges of current and pressure over which it was possible to make observations were rather limited. The following table contains some of the results obtained :—

p.	n.	H.	np/H.
6·5	220	43	33
6·5	480	91	34
6·5	670	133	33
6·5	780	164	31
9·1	370	107	31
9·1	475	133	33
			Mean 32·5

Thus, for hydrogen, $n = 32{\cdot}5\ \mathrm{H}/p$, and so is about 13 times greater than in nitrogen. The velocity seems, therefore, to be inversely as the density of the gas.

The numbers given above refer to the number of revolutions per second made by the discharge. For purposes of theoretical calculation it is more convenient to express the results in terms of the velocity of motion of the discharge in centimetres per second. Since the mean radius of the discharge tube was 1·33 cm., the revolutions per second must be multiplied by $2\pi \times 1{\cdot}33 = 8{\cdot}36$. If u denotes the velocity, we have, then, for nitrogen, $u = 20{\cdot}9\ \mathrm{H}/p$; for air, $u = 23{\cdot}4\ \mathrm{H}/p$; and for hydrogen, $u = 272\ \mathrm{H}/p$.

Theory of the Rotation of the Discharge.

The quantity observed was the velocity of rotation of the positive column of the discharge. In many of the experiments the negative glow covered the whole surface of the negative electrode and so could not be observed to rotate. Very often the positive column was striated when at rest, and remained so when rotating without any change in its appearance as seen through the stroboscopic disc. To simplify the theory, we shall suppose the discharge to be uniform and to be moving perpendicular to itself in a uniform magnetic field of strength H. If C denote the current carried by the discharge, then the transverse force per centimetre acting on the discharge is HC. If we suppose that this force is balanced by the resistance to the motion of the positive and negative ions through the gas, we get $HC = u\,(A_1N_1+A_2N_2)$, where u denotes the transverse velocity of the discharge, A_1 and A_2 denote the resistances to the motion of one positive and one negative ion respectively when moving with unit velocity, and N_1 and N_2 denote the numbers of positive and negative ions present per centimetre of the discharge. If X denotes the electric intensity along the discharge, then

$$C = eX\,(k_1N_1+k_2N_2),$$

where e is the charge on each ion and k_1 and k_2 are the velocities of the ions due to unit electric intensity. If v_1 and v_2 denote the velocities of the ions along the discharge, then

$$Xe = v_1A_1 \quad \text{and} \quad Xe = v_2A_2,$$

also
$$v_1 = k_1X \quad \text{and} \quad v_2 = k_2X,$$

so that
$$A_1 = e/k_1 \quad \text{and} \quad A_2 = e/k_2.$$

Hence
$$HC = eu\,(N_1/k_1+N_2/k_2).$$

Therefore
$$HX\,(k_1N_1+k_2N_2) = u\,(N_1/k_1+N_2/k_2),$$

and, if we assume $N_1 = N_2$, we get

$$u = HXk_1k_2, \quad \text{or} \quad u = \frac{H}{X}v_1v_2.$$

The Hall effect in the positive column in several gases was measured by one of us in 1901,* and it was shown that if Z is the transverse electric intensity due to the Hall effect, then, theoretically, $Z = \frac{1}{2}HX\,(k_2-k_1)$, in which Z was found experimentally to be given by the equation $Z = DH/p$, where D is a constant and p the pressure in millimetres of mercury.

Since the velocity of motion of the discharge is given approximately by the

* 'Proceedings of the Cambridge Philosophical Society,' vol. 11, Parts IV and V.

equation $u = \mathrm{CH}/p$, it follows that the Hall effect should be proportional to the velocity of motion. We have—

$$u = \mathrm{C}\frac{\mathrm{H}}{p} = \mathrm{HX}k_1k_2 \quad \text{and} \quad \mathrm{Z} = \mathrm{D}\frac{\mathrm{H}}{p} = \tfrac{1}{2}\mathrm{HX}(k_2-k_1),$$

so that $$\mathrm{C} = p\mathrm{X}k_1k_2 \quad \text{and} \quad 2\mathrm{D} = p\mathrm{X}(k_2-k_1).$$

If X is known, these two equations enable k_2 and k_1 to be calculated.

Unfortunately, there is considerable uncertainty about the value of X. X falls off as the current density is increased; and the current density in the experiments on the rate of motion of the discharge was considerably greater than in the experiments on the Hall effect. Further, the measurements of the Hall effect were made at lower pressures than those on the velocity of rotation, and it is probable that the formula $\mathrm{Z} = \mathrm{DH}/p$ is only an approximation to the truth, so that extrapolation is not allowable. We have, therefore, only calculated k_2 and k_1 for each gas at the pressure intermediate between the lowest pressure at which the velocity was measured and the highest pressure at which the Hall effect was measured, so reducing the extrapolation as much as possible.

The following table contains the results of this calculation :—

Gas.	D.	C.	p. Millimetres of Hg.	X. Volts per cm.	k_2. cm. per sec. per volt/cm.	k_1. cm. per sec. per volt/cm.
Air …………	0·0248	23·4	3·8	68	19,700	450
Hydrogen …	0·0205	272·0	3·8	55	25,000	5000

If we assume that the velocity of the positive ion varies inversely as the pressure, we get, at 760 mm., for air, $k_1 = 2{\cdot}2$ cm./sec., and for hydrogen, $k_1 = 25$ cm./sec. According to Zeleny, the velocity of the positive ions produced in air at 760 mm. by Röntgen rays is 1·4 cm./sec. for air and 6·7 cm./sec. for hydrogen.

Mr. Aston* has calculated k_1 from the results of measurements on the cathode dark space at pressures below 0·5 mm., and his results, calculated to 760 mm., are 0·78 for air and 7·7 for hydrogen. It seems probable, therefore, that k_1p does not vary much with the pressure. The above results show that k_2 is much larger than k_1 at low pressures, which probably means that some of the negative ions are free electrons.

If the empirical formulæ obtained for the Hall effect and for the velocity of motion are both assumed to apply at pressures below 3·8 mm., and k_1 and

* 'Roy. Soc. Proc.,' A, vol. 79, p. 80.

k_2 calculated, it is found that k_1 is nearly independent of the pressure, while k_2 rises rapidly with diminishing pressure.

The highest pressure at which the Hall effect in air was measured was 2·9 mm., and the lowest pressure at which the velocity of motion was measured was 4·7 mm., so that for 3·8 mm. the calculation of k_2 and k_1 is fairly reliable, but it is certainly not justifiable to use the formula for the velocity of motion below this pressure, or that for the Hall effect above it.

In the case of hydrogen, the Hall effect was not measured above 1 mm., and the velocity not below 6·5 mm., so that very much weight cannot be attached to the values of k_2 and k_1 calculated for hydrogen at 3·8 mm.

If we assume that k_1 is inversely proportional to the pressure at all pressures, then, since $u = \mathrm{CH}/p = \mathrm{HX}k_1k_2$, we see that $k_2\mathrm{X}$ is a constant for all the pressures at which the velocity of motion was measured.

It appears, therefore, that in air between 4·7 and 12 mm., the velocity of the negative ions in the positive column, *i.e.*, $k_2\mathrm{X}$, does not vary much with the pressure.

At low pressures, k_1 is small compared to k_2, so that the Hall effect gives k_2 approximately equal to $2\mathrm{D}/p\mathrm{X}$, and $k_2\mathrm{X}$ varies inversely as the pressure.

At high pressures, X is probably nearly proportional to the pressure, while k_2 is inversely proportional to the pressure, so that $k_2\mathrm{X}$ is constant.

Most of the apparatus used in this investigation was purchased out of a grant of £500 given to the Wheatstone Laboratory at King's College by the Drapers' Company, to whom, therefore, we wish to acknowledge our indebtedness.